EXACT EQUATIONS
FOR VACUUM CORRELATORS IN FIELD THEORY

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Abstract

Stochastic quantization is used to derive exact equations, connecting multilocal field correlators in the φ^3 theory and gluodynamics. Perturbative expansion of the obtained equations in the lowest orders is presented.

1 Introduction

Exact determination of nonperturbative effects is one of the main problems in field theory nowadays. We suggest to look for them in exact equations (eqs.) for vacuum correlators, which we derive, using stochastic quantization method [1].

In QCD the nonperturbative correlators are an essential part of operator product expansion (OPE), QCD sum rules [2] and, more recently, of the method of vacuum correlators (MVC) [3]. In the latter correlators are used as a dynamical input, and one needs to obtain eqs. for them, starting from the lagrangian.

There exist in literature two types of exact eqs. First is Dyson–Schwinger eqs.[4], which, however, are not applicable to QCD in the confining phase, because of the lack of gauge invariance. The other one is Makeenko-Migdal eqs.[5], which are closer to our eqs., but written not for correlators, but for Wilson loops in the large N_c limit. We hope, that the eqs., found in this letter for correlators, can be used more directly for the purposes, stated above.

Sections 2 and 3 are devoted to the φ^3 theory, and in Section 4 an alternative approach is applied to gluodynamics.

2 φ^3 theory

From the Euclidean action

$$S=\int dx\left[\frac{1}{2}\left(\partial_\mu\varphi\right)^2+\frac{m^2}{2}\varphi^2-\frac{g}{3}\varphi^3\right]$$

in the stochastic quantization method one obtains the Langevin equation

$$\dot{\varphi}(x,t)+\Big(m^2-\partial^2-g\varphi(x,t)\Big)\varphi(x,t)=\eta(x,t)\ , \tag{1}$$

where $<\eta(x,t)\eta(x',t')> \quad = 2\delta(x-x')\delta(t-t')$. In the absence of constant classical solutions the retarded Green function of (1) is

$$\begin{aligned} G(x,y,t) &= \theta(t)<x|e^{-(m^2-\partial^2-g\varphi)t}|y> = \\ &= \theta(t)\int(Dz)_{xy}exp\left(-m^2t-\int\limits_0^t\frac{\dot{z}^2}{4}d\xi+g\int\limits_0^t\varphi(z(\xi),\xi)d\xi\right)\ , \end{aligned}$$

where

$$(Dz)_{xy}=\lim_{N\to\infty}\prod_{n=1}^N\frac{d^4z(n)}{(4\pi\varepsilon)^2},\qquad N\varepsilon=t,\qquad z(\xi=0)=y,\qquad z(\xi=t)=x\ .$$

Here we used the path integral Feynman–Schwinger representation. Thus, the solution of (1) has the form

$$\varphi(x,t)=\int\limits_0^t dt'\int dy(Dz)_{xy}K_z(t,t')F_z(t,t')\eta(y,t')\ ,$$

where

$$K_z(t,t')=\theta(t-t')exp\left(-m^2(t-t')-\int\limits_{t'}^t\frac{\dot{z}^2(\xi)}{4}d\xi\right)\ ,$$

$$F_z(t,t')=exp\ \left(g\int\limits_{t'}^t\varphi(z(\xi),\xi)d\xi\right)\ .$$

To obtain the system of eqs. for $<\varphi>,\ <\varphi\eta>,\ <\varphi\varphi>,\ <\varphi\varphi\eta>,...$ we use the generating functional

$$Z[J]=<F_z(t,t')F_{\bar z}(\bar t,\bar t')exp\Big(\int dudtJ(u,t)\eta(u,t)\Big)>$$

and apply to it cumulant expansion [6]:

$$Z[J]=exp\sum_{n=1}^\infty\frac{1}{n!}\ll\left(g\int\limits_{t'}^t\varphi(z(\xi),\xi)d\xi+g\int\limits_{\bar t'}^{\bar t}\varphi\Big(\bar z(\bar\xi),\bar\xi\Big)d\bar\xi+\int dudtJ(u,t)\eta(u,t)\right)^n\gg\ .$$

Then

$$\begin{aligned} <F_z(t,t')\eta(y,t')> &= \frac{\delta Z[J]}{\delta J(y,t')}\bigg|_{\substack{J=0\\F_{\bar z}(\bar t,\bar t')=1}} = \\ &= \left\{\sum_{n=1}^\infty\frac{1}{(n-1)!}\ll\left(g\int\limits_{t'}^t\varphi(z(\xi),\xi)d\xi\right)^{n-1}\eta(y,t')\gg\right\}<F_z(t,t')>\ , \end{aligned}$$

$$
< F_z(t,t')\eta(y,t')\eta(u,t) >= \frac{\delta^2 Z[J]}{\delta J(y,t')\delta J(u,t)}\Bigg|_{\substack{J=0\\ F_{\bar z}(\bar t,\bar t')=1}} =
$$

$$
= \Bigg\{ \sum_{n=2}^{\infty} \frac{1}{(n-2)!} \ll \left(g\int_{t'}^{t} \varphi(z(\xi),\xi)d\xi \right)^{n-2} \eta(y,t')\eta(u,t) \gg +
$$

$$
+ \left[\sum_{n=1}^{\infty} \frac{1}{(n-1)!} \ll \left(g\int_{t'}^{t} \varphi(z(\xi),\xi)d\xi \right)^{n-1} \eta(y,t') \gg \right] \cdot
$$

$$
\cdot \left[\sum_{k=1}^{\infty} \frac{1}{(k-1)!} \ll \left(g\int_{t'}^{t} \varphi(z(\xi),\xi)d\xi \right)^{k-1} \eta(u,t) \gg \right] \Bigg\} < F_z(t,t') >
$$

and so on. In the bilocal approximation (i.e. omitting all the cumulants higher than quadratic), one gets:

$$
< \varphi(x,t) > \; =
$$

$$
= g\int_0^t dt' \int dy (Dz)_{xy} K_z(t,t') \int_{t'}^{t} d\xi < \varphi(z(\xi),\xi)\eta(y,t') >< F_z(t,t') > \quad , \tag{2}
$$

$$
< \varphi(x,t)\eta(\bar x,\bar t) > \; =
$$

$$
= 2\int (Dz)_{x\bar x} K_z(t,\bar t) < F_z(t,\bar t) > + g^2 \int_0^t dt' \int dy (Dz)_{xy} K_z(t,t') \cdot
$$

$$
\cdot \int_{t'}^{t} d\xi < \varphi(z(\xi),\xi)\eta(\bar x,\bar t) > \int_{t'}^{t} d\xi' < \varphi(z(\xi'),\xi')\eta(\bar x,\bar t) >< F_z(t,t') > \quad , \tag{3}
$$

$$
< \varphi(x,t)\varphi(\bar x,\bar t) > \; =
$$

$$
= g^2 \int_0^t dt' \int_0^{\bar t} d\bar t' \int dy d\bar y (Dz)_{xy} (D\bar z)_{\bar x\bar y} K_z(t,t') K_{\bar z}(\bar t,\bar t') \cdot
$$

$$
\cdot \left[\int_{t'}^{t} d\xi < \varphi(z(\xi),\xi)\eta(y,t') > + \int_{\bar t}^{\bar t} d\xi' < \varphi(\bar z(\xi'),\xi')\eta(y,t') > \right] \cdot
$$

$$
\cdot \left[\int_{t'}^{t} d\xi < \varphi(z(\xi),\xi)\eta(\bar y,\bar t') > + \int_{\bar t'}^{\bar t} d\xi' < \varphi(\bar z(\xi'),\xi')\eta(\bar y,\bar t') > \right] \cdot
$$

$$
\cdot < F_z(t,t')F_{\bar z}(\bar t,\bar t') > + 2\int_0^t dt' \int dy (Dz)_{xy} (D\bar z)_{\bar x y} K_z(t,t') K_{\bar z}(\bar t,t') < F_z(t,t')F_{\bar z}(\bar t,t') >, \tag{4}
$$

where

$$
< F_z(t,t') > \; = \; = exp \left(g\int_{t'}^{t} d\xi < \varphi(z(\xi),\xi) > + \right.
$$

$$+\frac{g^2}{2}\int\limits_{t'}^{t} d\xi \int\limits_{t'}^{t} d\xi' \,(<\varphi(z(\xi),\xi)\varphi(z(\xi'),\xi')> - <\varphi(z(\xi),\xi)><\varphi(z(\xi'),\xi')>)\,)\ , \qquad (5)$$

$$<F_z(t,t')F_{\bar z}(\bar t,\bar t')> \;=\; exp\left\{ g\int\limits_{t'}^{t} d\xi <\varphi(z(\xi),\xi)> + \right.$$

$$+g\int\limits_{\bar t'}^{\bar t} d\bar\xi <\varphi(\bar z(\bar\xi),\bar\xi)> + \frac{g^2}{2}\left[\int\limits_{t'}^{t} d\xi \int\limits_{t'}^{t} d\xi' <\varphi(z(\xi),\xi)\varphi(z(\xi'),\xi')> + \right.$$

$$+\int\limits_{\bar t'}^{\bar t} d\bar\xi \int\limits_{\bar t'}^{\bar t} d\bar\xi' <\varphi(\bar z(\bar\xi),\bar\xi)\varphi(\bar z(\bar\xi'),\bar\xi')> +2\int\limits_{t'}^{t} d\xi \int\limits_{\bar t'}^{\bar t} d\bar\xi <\varphi(z(\xi),\xi)\varphi(\bar z(\bar\xi),\bar\xi)> -$$

$$\left.\left. -\left(\int\limits_{t'}^{t} d\xi <\varphi(z(\xi),\xi)> + \int\limits_{\bar t'}^{\bar t} d\bar\xi <\varphi(\bar z(\bar\xi),\bar\xi)>\right)^2\right]\right\} \ . \qquad (6)$$

Eqs.(2)-(6) is the minimal closed set of eqs. for $<\varphi>$, $<\varphi\eta>$, $<\varphi\varphi>$.

Note, that $<\varphi(x)\varphi(x')>_{vac} = \lim\limits_{\substack{t\to\infty \\ t'\to\infty \\ (t-t')fixed}} <\varphi(x,t)\varphi(x',t')>_\eta$. It means, that without only suppositions about the structure of the real vacuum, we obtained eqs. for physical correlators.

3 Perturbative expansion of eqs.(2)-(6)

Expanding the right hand size of eqs.(2)-(6) in powers of g, we get in the lowest order

$$<\varphi(x,t)>^{(0)} = 0\ ,$$

$$<\varphi(x,t)\eta(\bar x,\bar t)>^{(0)} = 2\int(Dz)_{x\bar x}K_z(t,\bar t) = -\frac{1}{8\pi^2(\bar t - t)^2}e^{\frac{(\bar x - x)^2}{4(\bar t - t)}+m^2(\bar t - t)}\ .$$

The last term on the right hand size of (4) can be written as

$$\int dy <x|KF|y><y|KF|\bar x> \;=\; <x|KFKF|\bar x>\ ,$$

and for $t=\bar t$ one obtains for this term

$$\int\limits_0^{2t} dt_1 (Dz)_{x\bar x}K_z(t_1,0) <\bar F_z(t_1,0)>\ ,$$

where

$$\bar F_z(t_1,0) = exp\left[\int\limits_0^{\frac{t_1}{2}} \varphi\left(z(\xi),\xi + t - \frac{t_1}{2}\right)d\xi + \int\limits_{\frac{t_1}{2}}^{t_1} \varphi\Big(z(\xi),\xi + t - t_1\Big)d\xi\right]\ . \qquad (7)$$

Note, that in the asymptotical regime, when one drops out the dependence on ξ in $\varphi(z(\xi),\xi)$, we have $\bar{F}_z \to F_z$. Using (7),

$$< \varphi(x,t)\varphi(\bar{x},t) >^{(0)}=$$

$$= \int_0^{2t} dt_1 \int (Dz)_{x\bar{x}} K_z(t_1,0) = \int \frac{dp}{(2\pi)^4} \frac{e^{ip(x-\bar{x})}(1-e^{-2(p^2+m^2)t})}{p^2+m^2} \ .$$

This expression tends to the propagator of a free boson in the limit $t \to \infty$.

In the next order we have

$$< \varphi(x,t) >^{(1)} \ = \ 2g \int_0^t dt' \int dy (Dz)_{xy} K_z(t,t') \int_{t'}^t d\xi \int (D\bar{z})_{z(\xi)y} K_{\bar{z}}(\xi,t') =$$

$$= 2g \int_0^t dt' \int dy (Dz)_{xz(\xi)} K_z(t,\xi)(Dz)_{z(\xi)y} K_z(\xi,t') dz(\xi) \int_{t'}^t d\xi \int \frac{dp}{(2\pi)^4} e^{ip(z(\xi)-y)-(p^2+m^2)(\xi-t')} =$$

$$= g \int \frac{dp}{(2\pi)^4} \frac{1}{m^2(p^2+m^2)} + \underline{\underline{0}}(e^{-m^2 t}) \ ,$$

that in the limit $t \to \infty$ corresponds to the lowest order tadpole diagram.

Keeping $< \varphi >^{(1)}$ in the last term on the right hand size of (4), we obtain

$$< \varphi(x,t)\varphi(\bar{x},t) >^{(1)}= \int_0^{2t} dt_1 \int \frac{dp}{(2\pi)^4} e^{ip(x-\bar{x})-\left(p^2+m^2-g<\varphi>^{(1)}\right)t_1} =$$

$$= \int \frac{dp}{(2\pi)^4} \frac{e^{ip(x-\bar{x})}}{p^2+m^2-g<\varphi>^{(1)}} + \underline{\underline{0}}(e^{2(g<\varphi>^{(1)}-m^2)t}) \ . \qquad (8)$$

Expanding (8) in powers of g, one obtains an infinite set of diagrams.

Note, that the unstability of vacuum in the φ^3 theory leads to divergence of (8) if $g < \varphi >^{(1)} > m^2$, so that the stochastic process has not limiting equilibrium.

4 Gluodynamics

We shall start from the Langevin equation

$$\dot{A}^a_\mu = (D_\lambda F_{\lambda\mu})^a - \eta^a_\mu, \qquad (9)$$

where $F^a_{\lambda\mu} = \partial_\lambda A^a_\mu - \partial_\mu A^a_\lambda + g f^{abc} A^b_\lambda A^c_\mu$, $(D_\lambda F_{\lambda\mu})^a = \partial_\lambda F^a_{\lambda\mu} + g f^{abc} A^b_\lambda F^c_{\lambda\mu}$, and the sign of η^a_μ is changed. Let's use Schwinger gauge [7] $A_\mu(x,t)(x-x_0)_\mu = 0$, in which $A_\mu(x,t) = \int_{x_0}^x dz_\nu \alpha(z,x) F_{\nu\mu}(z,t)$, where $\alpha(z,x) = \frac{(z-x_0)_\lambda (x-x_0)_\lambda}{(x-x_0)^2}$, x_0 is an arbitrary point (here and later in all the integrals of the type $\int_{x_0}^x dz_\nu$ the path of integration is a straight line), and introduce the generating functional

$$\Phi_\beta(t) = P \ exp \ ig \oint_c dx_\mu (\int_{x_0}^x dz_\nu \alpha(z,x) F_{\nu\mu}(z,t) + \beta n_\mu(x,t))$$

, where $n_\mu(x,t)=\int_0^t \eta_\mu(x,t')dt'$, C is some fixed closed contour.

According to (9),

$$tr\frac{\partial}{\partial t} < \Phi_\beta(t) > = igtr\oint_c du_\mu < \Phi_\beta(t)V_\mu(\beta,u,x_0,t) >, \tag{10}$$

where

$$V_\mu(\beta,u,x_0,t)=\Phi(x_0,u,t)(D_\lambda F_{\lambda\mu}(u,t)+(\beta-1)\eta_\mu(u,t))\Phi(u,x_0,t),$$

$$\Phi(u,x_0,t)=P\ exp\ ig\int_{x_0}^u A_\mu(z,t)dz_\mu.$$

Applying to both sides of (10) cumulant expansion, using the formula [6]

$$< e^A B > = < e^A > (< B > + \sum_{n=1}^{\infty}\frac{1}{n!} \ll A^n B \gg),$$

where A and B are two statistically dependent matrixes, and putting $\beta=1$, one obtains in the bilocal approximation

$$tr(\int_{x_0}^y dz_\lambda \alpha(z,y)\int_{x_0}^u dx_\rho \alpha(x,u)\frac{\partial}{\partial t} < F_{\lambda\nu}(z,x_0,t)F_{\rho\mu}(x,x_0,t) > + \int_{x_0}^y dz_\lambda\alpha(z,y)\int_0^t dt'\frac{\partial}{\partial t}\cdot$$

$$< F_{\lambda\nu}(z,x_0,t)\eta_\mu(u,x_0,t,t') > + \int_{x_0}^u dx_\rho\alpha(x,u)\int_0^t dt'\frac{\partial}{\partial t} < F_{\rho\mu}(x,x_0,t)\eta_\nu(y,x_0,t,t') > +$$

$$+\int_{x_0}^y dz_\lambda\alpha(z,y) < F_{\lambda\nu}(z,x_0,t)\eta_\mu(u,x_0,t,t) > +$$

$$+\int_{x_0}^u dx_\rho\alpha(x,u) < F_{\rho\mu}(x,x_0,t)\eta_\nu(y,x_0,t,t) > +$$

$$+\int_0^t dt'(< \eta_\nu(y,x_0,t,t)\eta_\mu(u,x_0,t,t') > + < \eta_\nu(y,x_0,t,t')\eta_\mu(u,x_0,t,t) >) + \int_0^t dt'\int_0^t dt''\cdot$$

$$\cdot\frac{\partial}{\partial t} < \eta_\nu(y,x_0,t,t')\eta_\mu(u,x_0,t,t'') >) = 2tr(\int_{x_0}^y dz_\lambda\alpha(z,y)\frac{\partial}{\partial u_\rho} < F_{\lambda\nu}(z,x_0,t)F_{\rho\mu}(u,x_0,t) > +$$

$$+\int_0^t dt'\frac{\partial}{\partial u_\rho} < F_{\rho\mu}(u,x_0,t)\eta_\nu(y,x_0,t,t') >), \tag{11}$$

where

$$F_{\lambda\nu}(z,x_0,t)=\Phi(x_0,z,t)F_{\lambda\nu}(z,t)\Phi(z,x_0,t), \eta_\mu(u,x_0,t,t')=\Phi(x_0,u,t)\cdot$$

$$\cdot\eta_\mu(u,t')\Phi(u,x_0,t).$$

Differentiating (10) by β, we get

$$tr\frac{\partial}{\partial t} < \Phi_\beta(t)n_\mu(u,x_0,t) > = tr(ig\oint_c dz_\nu < \Phi_\beta(t)n_\nu(z,x_0,t)V_\mu(\beta,u,x_0,t) > + \tag{12}$$

$$+ < \Phi_\beta(t)\eta_\mu(u,x_0,t) >),$$

that in bilocal approximation yields

$$tr\frac{\partial}{\partial t}(\int_{x_0}^{y} dz_\lambda \alpha(z,y) < F_{\lambda\nu}(z,x_0,t) n_\mu(u,x_0,t) > + < n_\nu(y,x_0,t) n_\mu(u,x_0,t) >) =$$

$$= tr(< \Phi(x_0,u,t)(D_\lambda F_{\lambda\mu}(u,t))\Phi(u,x_0,t) n_\nu(y,x_0,t) > + \int_{x_0}^{y} dz_\lambda \alpha(z,y) < F_{\lambda\nu}(z,x_0,t)\cdot$$

$$\cdot \eta_\mu(u,x_0,t) > + < n_\nu(y,x_0,t)\eta_\mu(u,x_0,t) >).$$

Noticing that

$$tr(< \Phi(x_0,u,t)(D_\lambda F_{\lambda\mu}(u,t))\Phi(u,x_0,t) n_\nu(y,x_0,t) >=$$

$$= \frac{\partial}{\partial u_\lambda} tr < F_{\lambda\mu}(u,x_0,t) n_\nu(y,x_0,t) > + ig \int_{x_0}^{u} dx_\rho \cdot$$

$$\cdot \alpha(x,u)(tr < F_{\lambda\mu}(u,x_0,t) n_\nu(y,x_0,t) F_{\lambda\rho}(x,x_0,t) > - tr < F_{\lambda\mu}(u,x_0,t) F_{\lambda\rho}(x,x_0,t)\cdot$$

$\cdot n_\nu(y,x_0,t) \ \ >)$ and using the expression for threelocal path–ordered cumulant $\ll 123 \gg = < 123 > - < 1 >< 23 > - < 12 >< 3 > + < 1 >< 2 >< 3 >$ [7], one obtains

$$tr(\int_{x_0}^{y} dz_\lambda \alpha(z,y)\frac{\partial}{\partial t} < F_{\lambda\nu}(z,x_0,t)\eta_\mu(u,x_0,t,t') > + \int_0^t dt'' \frac{\partial}{\partial t} < \eta_\nu(y,x_0,t')\eta_\mu(u,x_0,t,t'') > -$$

$$- \frac{\partial}{\partial u_\lambda} < F_{\lambda\mu}(u,x_0,t)\eta_\nu(y,x_0,t,t') >) = -tr < \eta_\nu(y,x_0,t,t)\eta_\mu(u,x_0,t,t') > . \qquad (13)$$

Differentiating (12) by β, putting $\beta = 1$ and using the expression for fourlocal path–ordered cumulant $\ll 1234 \gg = < 1234 > - \ll 123 \gg < 4 > - < 1 > \ll 234 \gg - \ll 12 \gg \ll 34 \gg - \ll 12 \gg < 3 >< 4 > - < 1 > \ll 23 \gg < 4 > - < 1 >< 2 > \ll 34 \gg - < 1 >< 2 >< 3 >< 4 >$, we have in bilocal approximation:

$$tr(\int_0^t dt'' \frac{\partial}{\partial t} < \eta_\nu(y,x_0,t,t')\eta_\mu(u,x_0,t,t'') > + < \eta_\nu(y,x_0,t,t)\eta_\mu(u,x_0,t,t') > -$$

$$- < \eta_\nu(y,x_0,t,t')\eta_\mu(u,x_0,t,t) >) = g^2 tr(\int_{x_0}^{u} dx_\sigma \alpha(x,u) < F_{\rho\mu}(u,x_0,t) F_{\rho\sigma}(x,x_0,t) > \cdot$$

$$\cdot \oint_c dz_\lambda \int_0^t dt'' < \eta_\lambda(z,x_0,t,t')\eta_\nu(y,x_0,t,t'') > - \oint_c dz_\lambda \int_0^t dt'' < F_{\rho\mu}(u,x_0,t)\eta_\lambda(z,x_0,t,t') > \cdot$$

$$\cdot \int_{x_0}^{u} dx_\sigma \alpha(x,u) < \eta_\nu(y,x_0,t,t'') F_{\rho\sigma}(x,x_0,t) >) \qquad (14)$$

Eqs. (11),(13) and (14) is the minimal closed set of eqs. for correlators

$$< F_{\lambda\nu}(z,x_0,t) F_{\rho\mu}(x,x_0,t) >, < F_{\lambda\nu}(z,x_0,t)\eta_\mu(u,x_0,t,t') >,$$

$$< \eta_\nu(y,x_0,t,t')\eta_\mu(u,x_0,t,t'') > .$$

Note, that beyond obtained eqs. (and all other eqs., one is able to get, using Φ_β) there are some additional relations, connecting gauge–invariant correlators. Let $G_{\mu_1 \ldots \mu_n}(x_1,t_1,...,x_n,t_n,x_0,t)$ be a product of some number of $F_{\mu\nu}$ and (or) η_μ, which

are given in the points $x_1, ..., x_n$ at the moments $t_1, ..., t_n$ of fictitious time respectively, and all the parallel transporters between x_0 and each of these points are given at the same moment t. Then, using nonabelian Bianchi identities $D_\lambda \tilde{F}_{\lambda\mu} = 0$, we have [7]:

$$tr \frac{\partial}{\partial x_\lambda} < \tilde{F}_{\lambda\mu}(x, x_0, t') G_{\mu_1...\mu_n} >= igtr \int_{x_0}^{x} dz_\rho \alpha(z, x) (< \tilde{F}_{\lambda\mu}(x, x_0, t') F_{\lambda\rho}(z, x_0, t) G_{\mu_1...\mu_n} > -$$

$$- < \tilde{F}_{\lambda\mu}(x, x_0, t') G_{\mu_1...\mu_n} F_{\lambda\rho}(z, x_0, t) >), \quad \text{where} \;\; x \neq x_1, ..., x \neq x_n.$$

To check ourselfs, let's consider eqs. (11), (13) and (14) in the order g^0. Looking for $< A^a_\nu(y, t) \eta^b_\mu(u, t') >$ in the form $\delta^{ab} d_{\mu\nu}(z, \tau)$, where
$z = u - y, \tau = |t - t'|, d_{\nu\mu}(z, \tau) = d_{\mu\nu}(z, \tau), d_{\mu\nu}(-z, \tau) = d_{\mu\nu}(z, \tau)$, one obtains from (13):
$(\delta_{\mu\lambda} \frac{\partial}{\partial \tau} + \frac{\partial^2}{\partial z_\mu \partial z_\lambda} - \delta_{\mu\lambda} \partial^2) d_{\lambda\nu}(z, \tau) = -2\delta_{\mu\nu} \delta(z) \delta(\tau)$ and, thus,
$d_{\mu\nu}(k, \tau) = -2\theta(\tau)(T_{\mu\nu} e^{-k^2\tau} + L_{\mu\nu})$, where $T_{\mu\nu} = \delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}, L_{\mu\nu} = \frac{k_\mu k_\nu}{k^2}$.

Looking for $< A^a_\nu(y, t) A^b_\mu(u, t) >$ in the form $\delta^{ab} h_{\mu\nu}(z, t)$, where
$h_{\nu\mu}(z, t) = h_{\mu\nu}(z, t), h_{\mu\nu}(-z, t) = h_{\mu\nu}(z, t)$, we have from (11):
$(\frac{1}{2} \delta_{\mu\lambda} \frac{\partial}{\partial t} + \frac{\partial^2}{\partial z_\mu \partial z_\lambda} - \delta_{\mu\lambda} \partial^2) h_{\lambda\nu}(z, t) = -d_{\mu\nu}(z, 0)$.
So, $h_{\mu\nu}(k, t) = \frac{1}{k^2} T_{\mu\nu}(1 - e^{-2k^2 t}) + 2t L_{\mu\nu}$, that is the ordinary photon propagator in the stochastic quantization method [1].

5 Conclusion

The main result of the present letter is eqs. (11), (13) and (14). They are explicitly gauge invariant and produce correct perturbative results in the lowest order.

The principle of separation of perturbative and nonperturbative contributions in the obtained eqs., introduction of quarks and the problem of regularization will be treated in the next publications.

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